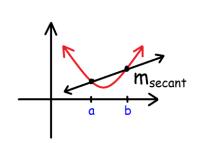
Applications of Derivatives - Formula Sheet:

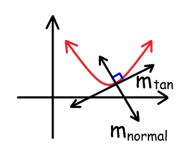


Average Rate of Change:

$$m_{secant} = \frac{f(b) - f(a)}{b - a}$$

Tangent Line Equation:

$$y - y_1 = m(x - x_1)$$

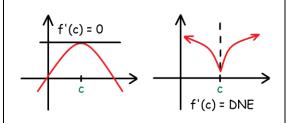


Instantaneous Rate of Change:

$$m_{tangent} = f'(c)$$

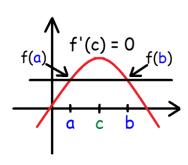
The Normal Line:

$$m_{normal} = -\frac{1}{m_{tangent}}$$



Critical Points:

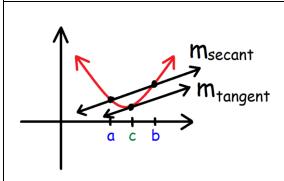
$$f'(c) = 0$$
 or $f'(c) = DNE$



Rolle's Theorem:

- 1. f(x) is continuous on [a, b]
- 2. f(x) is differentiable on (a, b)
- 3. f(a) = f(b)

If the 3 conditions above are met, then there is a number c in (a, b) where f'(c) = 0.



Mean Value Theorem:

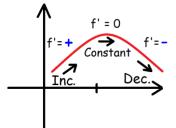
If f(x) is continuous on [a, b] and differentiable on (a, b), then there is a number c in (a, b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

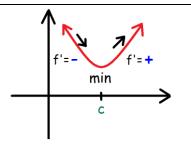
$$m_{tangent} = m_{secant}$$

Increasing/Decreasing Test:

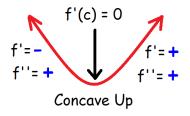
- 1. f(x) is increasing when f'(x) > 0.
- 2. f(x) is decreasing when f'(x) < 0.
- 3. If f'(x) does not change sign at c, then f(x) has no relative minimum or maximum at c.



First Derivative Test:

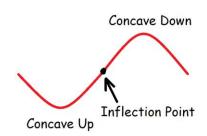


- 1. If f' changes from + to at c, then f has a local max. at c.
- 2. If f' changes from to + at c, then f has a local min. at c.
- 3. If f' doesn't change sign at c, then f has no local min. or max. at c.



Concavity Test:

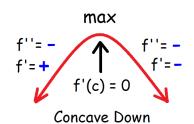
- 1. f is concave up when f''(x) > 0 or when f'(x) is increasing.
- 2. f is concave down when f''(x) < 0 or when f'(x) is decreasing.



Inflection Points:

If f''(c) = 0 or f''(c) does not exist, then there is an inflection point at c if the concavity changes at c.

Note: f must be continuous near c.



Second Derivative Test:

1. If f'(c) = 0 and f''(c) > 0, then f has a relative minimum at c. 2. If f'(c) = 0 and f''(c) < 0, then f has a relative maximum at c.

Note: f'' must be continuous near c.

L'hospital's Rule:

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

Note: $g'(x) \neq 0$ near x = a.

Newton's Method for Approximating the Zeros of a Function:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Differentials:

$$dy = f'(x) dx$$

 $dx \rightarrow Differential of x$

dy → Differential of y

Note: $\Delta x = dx$ and $\Delta y \approx dy$

$$\Delta \mathbf{y} = y_2 - y_1 = f(a + \Delta x) - f(a)$$

Tangent Line Approximations: Point(a, f(a))

$$y = f(a) + f'(a)(x - a)$$

Linear Equation – Point Slope Form: $Point(x_1, y_1)$

$$y - y_1 = m(x - x_1)$$

Note: m = f'(a) $a = x_1$ $f(a) = y_1$

Description:

 $C(x) \rightarrow Cost Function$

 $c(x) \rightarrow Average Cost Function$

 $C'(x) \rightarrow Marginal Cost Function$

Note: c(x) is a minimum when C'(x) = c(x)

Average Cost Function:

$$c(x) = \frac{C(x)}{x}$$

Revenue Function:

$$R(x) = x \cdot p(x)$$

Note: p(x) *is the price* (*demand*) *function*.

Profit Function:

$$P(x) = R(x) - C(x)$$

Note: Max profit occurs when R'(x) = C'(x)

Description:

 $R'(x) \rightarrow Marginal Revenue$

P'(x) → Marginal Profit

 $C'(x) \rightarrow Marginal Cost$

Marginal Profit:

$$\mathbf{P}'(\mathbf{x}) = R'(\mathbf{x}) - C'(\mathbf{x})$$

Average Velocity:

$$\bar{v} = \frac{s(b) - s(a)}{b - a}$$

Average Acceleration:

$$\bar{a} = \frac{v(b) - v(a)}{b - a}$$

Instantaneous Velocity:

$$v(t) = s'(t)$$

Instantaneous Acceleration:

$$a(t) = v'(t)$$

Displacement:

$$d = s(b) - s(a)$$

The Position Function:

s(t)