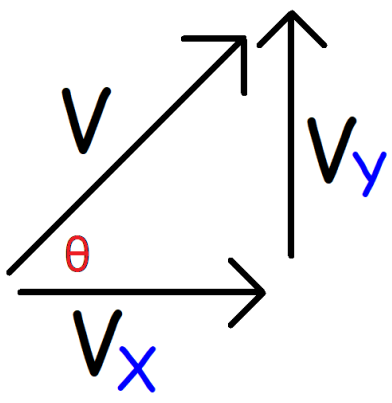
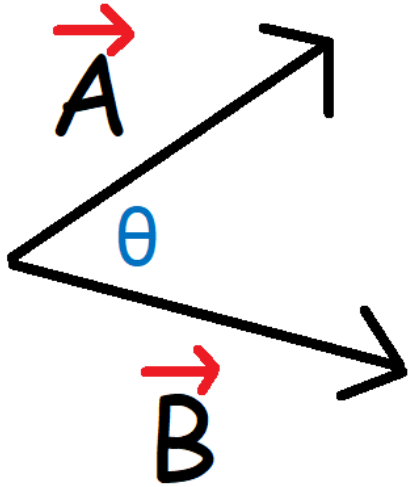
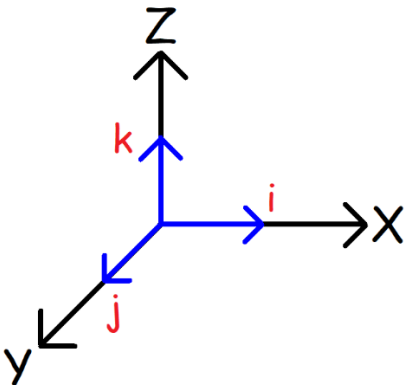
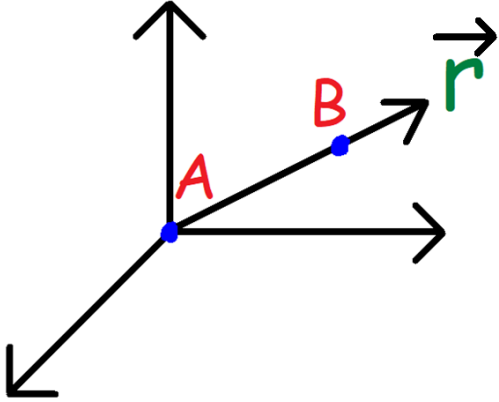
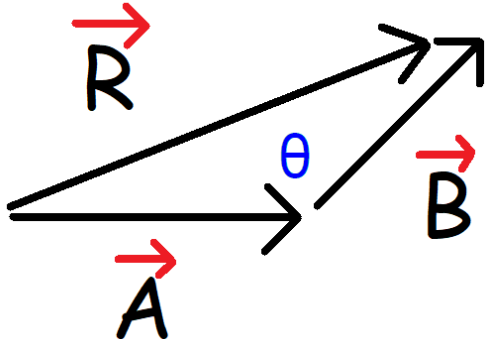
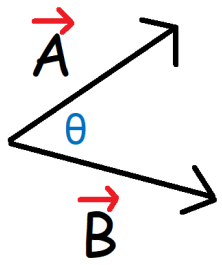
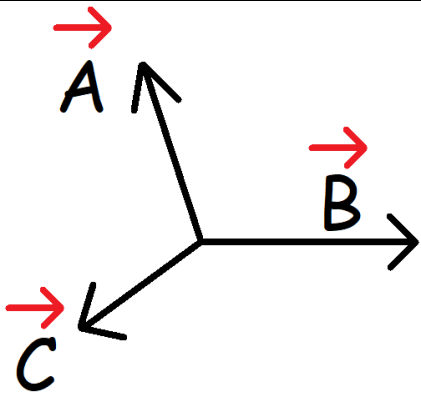
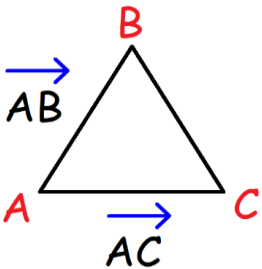
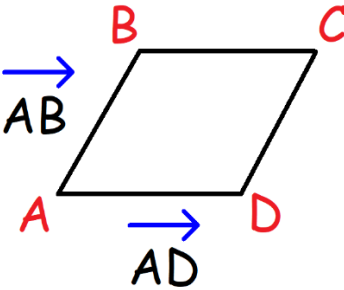
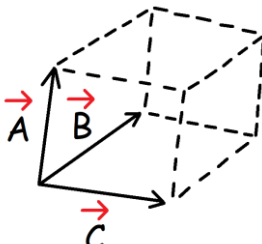
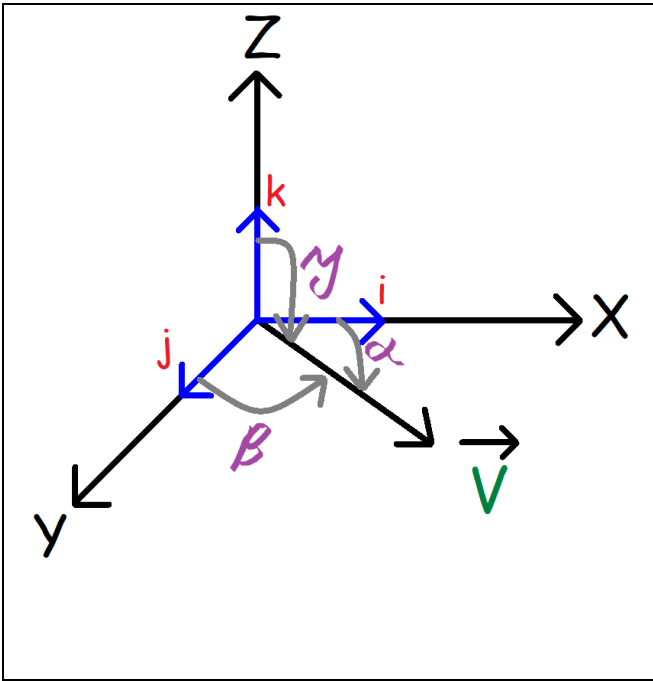


Vectors – Formula Sheet:

	<p>2D Vectors:</p> $V_x = V \cos(\theta)$ $V_y = V \sin(\theta)$ $\theta = \tan^{-1}\left(\frac{V_y}{V_x}\right)$ $ V = \sqrt{V_x^2 + V_y^2}$ $\vec{V} = V \cos(\theta) \mathbf{i} + V \sin(\theta) \mathbf{j}$
	<p>Scalar Dot Product:</p> $\vec{A} \cdot \vec{B} = A B \cos(\theta) = A_x B_x + A_y B_y + A_z B_z$ $ A = \sqrt{A_x^2 + A_y^2 + A_z^2}$ $ B = \sqrt{B_x^2 + B_y^2 + B_z^2}$ <p>Vector Cross Product:</p> $\vec{A} \times \vec{B} = A B \sin(\theta) \hat{r} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$ $\vec{A} \times \vec{B} = (A_y B_z - A_z B_y) \mathbf{i} + (A_z B_x - A_x B_z) \mathbf{j} + (A_x B_y - A_y B_x) \mathbf{k}$
$\vec{F} = F \vec{U}$ <div style="display: flex; justify-content: space-around; margin-top: 5px;"> <div style="text-align: center;"> \uparrow Vector </div> <div style="text-align: center;"> \uparrow Magnitude </div> <div style="text-align: center;"> \uparrow Direction </div> </div>	<p>Unit Vector – A vector with a magnitude of 1 that is used to give direction to another vector.</p> $\vec{U} = \frac{\vec{V}}{ V }$
	<p>Standard Unit Vectors:</p> $\begin{aligned} \mathbf{i} &= \langle 1, 0, 0 \rangle & \mathbf{i} \cdot \mathbf{i} &= 1 & \mathbf{i} \cdot \mathbf{j} &= 0 \\ \mathbf{j} &= \langle 0, 1, 0 \rangle & \mathbf{j} \cdot \mathbf{j} &= 1 & \mathbf{j} \cdot \mathbf{k} &= 0 \\ \mathbf{k} &= \langle 0, 0, 1 \rangle & \mathbf{k} \cdot \mathbf{k} &= 1 & \mathbf{i} \cdot \mathbf{k} &= 0 \end{aligned}$ $\begin{aligned} \mathbf{i} \times \mathbf{j} &= \mathbf{k} & \mathbf{j} \times \mathbf{i} &= -\mathbf{k} & \mathbf{i} \times \mathbf{i} &= 0 \\ \mathbf{j} \times \mathbf{k} &= \mathbf{i} & \mathbf{k} \times \mathbf{j} &= -\mathbf{i} & \mathbf{j} \times \mathbf{j} &= 0 \\ \mathbf{k} \times \mathbf{i} &= \mathbf{j} & \mathbf{i} \times \mathbf{k} &= -\mathbf{j} & \mathbf{k} \times \mathbf{k} &= 0 \end{aligned}$

 <p>A 3D coordinate system with three axes. Point A is at the origin. Point B is in the first octant. A vector \vec{r} (green) points from A to B. The components of \vec{r} are r_x, r_y, and r_z.</p>	<p>Position Vector:</p> $\vec{r}_{AB} = (X_B - X_A)\mathbf{i} + (Y_B - Y_A)\mathbf{j} + (Z_B - Z_A)\mathbf{k}$ $\vec{r}_{AB} = r_x\mathbf{i} + r_y\mathbf{j} + r_z\mathbf{k}$ $r_x = X_B - X_A \quad r_y = Y_B - Y_A \quad r_z = Z_B - Z_A$ $ \vec{r}_{AB} = \sqrt{r_x^2 + r_y^2 + r_z^2}$ $\hat{r} = \frac{\vec{r}_{AB}}{ \vec{r}_{AB} } = \frac{r_x\mathbf{i} + r_y\mathbf{j} + r_z\mathbf{k}}{\sqrt{r_x^2 + r_y^2 + r_z^2}}$
 <p>Two vectors \vec{A} and \vec{B} are shown. Vector \vec{B} is added to \vec{A} to form the resultant vector \vec{R}. The angle between \vec{A} and \vec{B} is θ.</p>	<p>Vector Addition:</p> $\vec{R} = \vec{A} + \vec{B}$ $\vec{R} = (A_x + B_x)\mathbf{i} + (A_y + B_y)\mathbf{j} + (A_z + B_z)\mathbf{k}$ $ \vec{R} = \sqrt{ A ^2 + B ^2 + 2 A B \cos(\theta)}$
 <p>Two vectors \vec{A} and \vec{B} originate from the same point. The angle between them is θ.</p>	<p>Finding The Angle Between Two Vectors:</p> $\theta = \cos^{-1}\left(\frac{\vec{A} \cdot \vec{B}}{ A B }\right)$ $\theta = \cos^{-1}\left(\frac{A_x B_x + A_y B_y + A_z B_z}{\sqrt{A_x^2 + A_y^2 + A_z^2} \times \sqrt{B_x^2 + B_y^2 + B_z^2}}\right)$
<p>Scalar Projection:</p> $\text{Comp}_{\vec{a}}\vec{b} = \frac{\vec{a} \cdot \vec{b}}{ a } \quad \text{"b onto a"}$ $\text{Comp}_{\vec{b}}\vec{a} = \frac{\vec{a} \cdot \vec{b}}{ b } \quad \text{"a onto b"}$	<p>Vector Projection:</p> $\text{Proj}_{\vec{a}}\vec{b} = \frac{\vec{a} \cdot \vec{b}}{ a ^2} \cdot \vec{a} \quad \text{"b onto a"}$ $\text{Proj}_{\vec{b}}\vec{a} = \frac{\vec{a} \cdot \vec{b}}{ b ^2} \cdot \vec{b} \quad \text{"a onto b"}$

	<p>Scalar Triple Product:</p> $\vec{A} \cdot (\vec{B} \times \vec{C}) = \det \begin{vmatrix} A_X & A_Y & A_Z \\ B_X & B_Y & B_Z \\ C_X & C_Y & C_Z \end{vmatrix}$ $= (A_Y B_Z - A_Z B_Y) C_X + (A_Z B_X - A_X B_Z) C_Y + (A_X B_Y - A_Y B_X) C_Z$ $\vec{A} \cdot (\vec{B} \times \vec{C}) = \vec{B} \cdot (\vec{C} \times \vec{A}) = \vec{C} \cdot (\vec{A} \times \vec{B})$ $\vec{A} \cdot (\vec{B} \times \vec{C}) = -\vec{A} \cdot (\vec{C} \times \vec{B})$ <p>Vector Triple Product:</p> $\vec{A} \times (\vec{B} \times \vec{C}) = (\vec{A} \cdot \vec{C}) \vec{B} - (\vec{A} \cdot \vec{B}) \vec{C}$
	<p>Area of a Triangle:</p> $Area = \frac{1}{2} \vec{AB} \times \vec{AC} $
	<p>Area of a Parallelogram:</p> $Area = \vec{AB} \times \vec{AD} $
	<p>Volume of a Parallelepiped: (Triple Scalar Product)</p> $Volume = \vec{A} \cdot (\vec{B} \times \vec{C}) $



Direction Angles:

$$\cos(\alpha) = \frac{V_x}{|V|} \quad \cos(\beta) = \frac{V_y}{|V|} \quad \cos(\gamma) = \frac{V_z}{|V|}$$

$$\vec{V} = |V| \cos(\alpha) \mathbf{i} + |V| \cos(\beta) \mathbf{j} + |V| \cos(\gamma) \mathbf{k}$$

$$\cos^2(\alpha) + \cos^2(\beta) + \cos^2(\gamma) = 1$$

$$\hat{V} = U_V = \frac{V_x}{|V|} \mathbf{i} + \frac{V_y}{|V|} \mathbf{j} + \frac{V_z}{|V|} \mathbf{k}$$

$$\hat{V} = U_V = \cos(\alpha) \mathbf{i} + \cos(\beta) \mathbf{j} + \cos(\gamma) \mathbf{k}$$