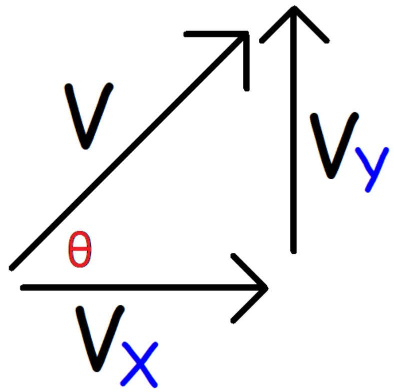
Vectors – Formula Sheet:



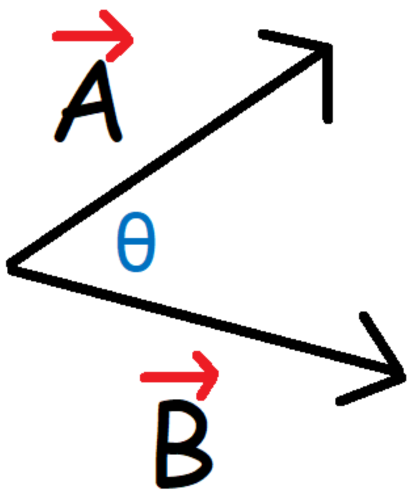
2D Vectors:

$$\boldsymbol{V_X} = V\cos(\theta)$$
$$\boldsymbol{V_Y} = V\sin(\theta)$$

$$\boldsymbol{\theta} = \tan^{-1}\left(\frac{V_Y}{V_X}\right)$$

$$|\boldsymbol{V}| = \sqrt{V_X^2 + V_Y^2}$$

$$\vec{\boldsymbol{V}} = |V|\cos(\theta)\boldsymbol{i} + |V|\sin(\theta)\boldsymbol{j}$$



Scalar Dot Product:

$$\vec{\boldsymbol{A}} \cdot \vec{\boldsymbol{B}} = |A||B|\cos(\theta) = A_X B_X + A_Y B_Y + A_Z B_Z$$

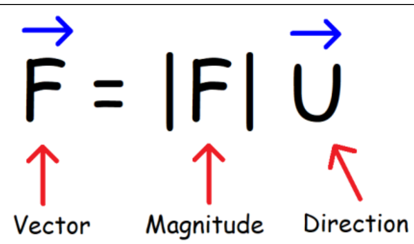
$$|\boldsymbol{A}| = \sqrt{A_X^2 + A_Y^2 + A_Z^2}$$

$$|\boldsymbol{B}| = \sqrt{B_X^2 + B_Y^2 + B_Z^2}$$

Vector Cross Product:

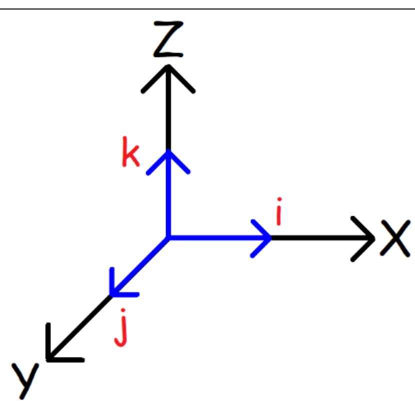
$$\vec{\boldsymbol{A}}\, x \vec{\boldsymbol{B}} = |A|\ |B|\sin(\theta)\hat{\boldsymbol{r}} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ A_X & A_Y & A_Z \\ B_X & B_Y & B_Z \end{vmatrix}$$

$$\vec{\boldsymbol{A}} x \vec{\boldsymbol{B}} = (A_Y B_Z - A_Z B_Y)\boldsymbol{i} + (A_Z B_X - A_X B_Z)\boldsymbol{j} + (A_X B_Y - A_Y B_X)\boldsymbol{k}$$



Unit Vector – A vector with a magnitude of 1 that is used to give direction to another vector.

$$\vec{\boldsymbol{U}} = \frac{\vec{V}}{|V|}$$



Standard Unit Vectors:

$$i = < 1, 0, 0 > \qquad i \cdot i = 1 \qquad i \cdot j = 0$$
$$j = < 0,1,0 > \qquad j \cdot j = 1 \qquad j \cdot k = 0$$
$$k = < 0,0,1 > \qquad k \cdot k = 1 \qquad i \cdot k = 0$$

$$i\ x\ j = k \qquad j\ x\ i = -k \qquad i\ x\ i = 0$$
$$j\ x\ k = i \qquad k\ x\ j = -i \qquad j\ x\ j = 0$$
$$k\ x\ i = j \qquad i\ x\ k = -j \qquad k\ x\ k = 0$$

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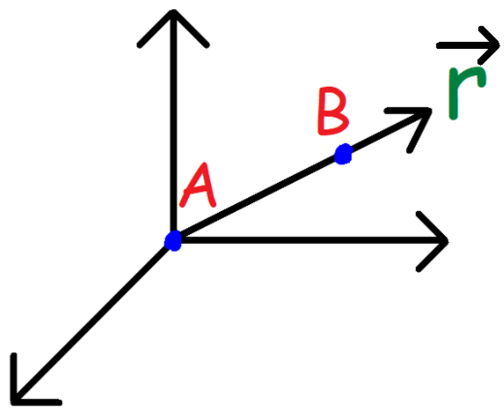


Position Vector:

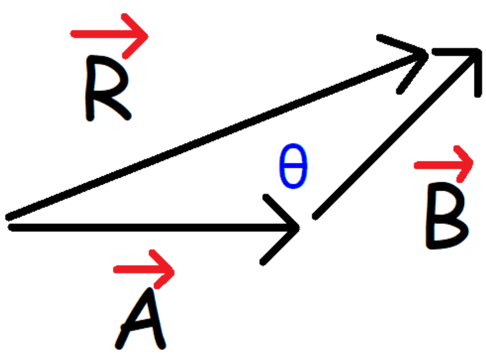
$$\overrightarrow{\boldsymbol{r_{AB}}} = (X_B - X_A)\boldsymbol{i} + (Y_B - Y_A)\boldsymbol{j} + (Z_B - Z_A)\boldsymbol{k}$$

$$\overrightarrow{\boldsymbol{r_{AB}}} = r_x\boldsymbol{i} + r_y\boldsymbol{j} + r_z\boldsymbol{k}$$

$$r_x = X_B - X_A \qquad r_y = Y_B - Y_A \qquad r_z = Z_B - Z_A$$

$$|\overrightarrow{\boldsymbol{r_{AB}}}| = \sqrt{r_x^2 + r_y^2 + r_z^2}$$

$$\hat{\boldsymbol{r}} = \frac{\overrightarrow{r_{AB}}}{|r|} = \frac{r_x\boldsymbol{i} + r_y\boldsymbol{j} + r_z\boldsymbol{k}}{\sqrt{r_x^2 + r_y^2 + r_z^2}}$$

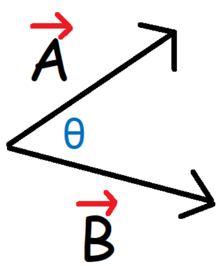


Vector Addition:

$$\vec{\boldsymbol{R}} = \vec{A} + \vec{B}$$

$$\vec{\boldsymbol{R}} = (A_X + B_X)\boldsymbol{i} + (A_Y + B_Y)\boldsymbol{j} + (A_Z + B_Z)\boldsymbol{k}$$

$$|\boldsymbol{R}| = \sqrt{|A|^2 + |B|^2 + 2|A||B|cos(\theta)}$$



Finding The Angle Between Two Vectors:

$$\boldsymbol{\theta} = cos^{-1}\left(\frac{\vec{A} \cdot \vec{B}}{|A|\,|B|}\right)$$

$$\boldsymbol{\theta} = cos^{-1}\left(\frac{A_X B_X + A_Y B_Y + A_Z B_Z}{\sqrt{A_x^2 + A_Y^2 + A_Z^2} \times \sqrt{B_X^2 + B_Y^2 + B_Z^2}}\right)$$

Scalar Projection:

$$\boldsymbol{Comp_{\vec{a}}\vec{b}} = \frac{\vec{a} \cdot \vec{b}}{|a|} \qquad \text{"b onto a"}$$

$$\boldsymbol{Comp_{\vec{b}}\vec{a}} = \frac{\vec{a} \cdot \vec{b}}{|b|} \qquad \text{"a onto b"}$$

Vector Projection:

$$\boldsymbol{Proj_{\vec{a}}\vec{b}} = \frac{\vec{a} \cdot \vec{b}}{|a|^2} \cdot \vec{a} \qquad \text{"b onto a"}$$

$$\boldsymbol{Proj_{\vec{b}}\vec{a}} = \frac{\vec{a} \cdot \vec{b}}{|b|^2} \cdot \vec{b} \qquad \text{"a onto b"}$$

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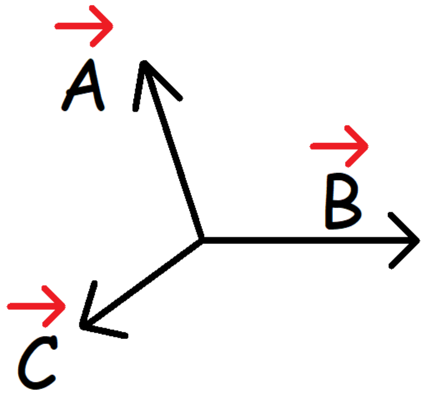


Scalar Triple Product:

$$\vec{\boldsymbol{A}} \cdot (\vec{\boldsymbol{B}}\, \boldsymbol{x}\, \vec{\boldsymbol{C}}) = det \begin{vmatrix} A_X & A_Y & A_Z \\ B_X & B_Y & B_Z \\ C_X & C_Y & C_Z \end{vmatrix}$$

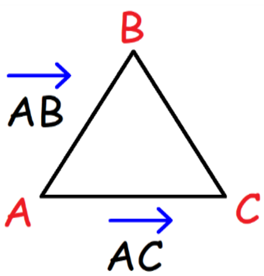
$$= (A_Y B_Z - A_Z B_Y)C_X + (A_Z B_X - A_X B_Z)C_Y + (A_X B_Y - A_Y B_X)C_Z$$

$$\vec{\boldsymbol{A}} \cdot (\vec{\boldsymbol{B}}\, \boldsymbol{x}\, \vec{\boldsymbol{C}}) = \vec{B} \cdot (\vec{C}\ x\ \vec{A}) = \vec{C} \cdot (\vec{A}\ x\ \vec{B})$$

$$\vec{\boldsymbol{A}} \cdot (\vec{\boldsymbol{B}}\, \boldsymbol{x}\, \vec{\boldsymbol{C}}) = -\vec{A} \cdot (\vec{C}\ x\ \vec{B})$$

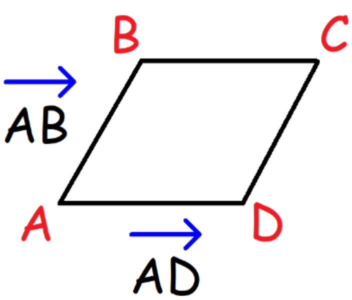
Vector Triple Product:

$$\vec{\boldsymbol{A}}\, \boldsymbol{x}\, (\vec{\boldsymbol{B}}\, \boldsymbol{x}\, \vec{\boldsymbol{C}}) = (\vec{A} \cdot \vec{C})\vec{B} - (\vec{A} \cdot \vec{B})\vec{C}$$



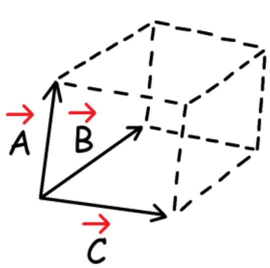
Area of a Triangle:

$$\boldsymbol{Area} = \frac{1}{2}\left|\overrightarrow{AB}\ x\ \overrightarrow{AC}\right|$$



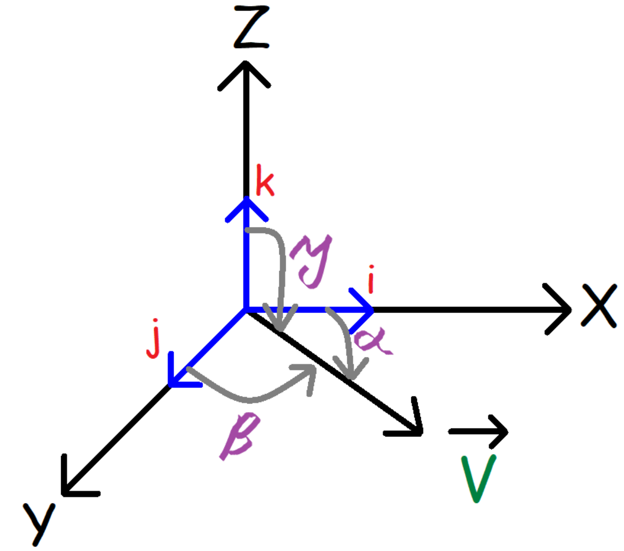
Area of a Parallelogram:

$$\boldsymbol{Area} = \left|\overrightarrow{AB}\ x\ \overrightarrow{AD}\right|$$



Volume of a Parallelepiped: **(Triple Scalar Product)**

$$\boldsymbol{Volume} = \left|\vec{A} \cdot (\vec{B}\, x\, \vec{C})\right|$$



Direction Angles:

$$cos(\alpha) = \frac{V_X}{|V|} \quad \cos(\beta) = \frac{V_Y}{|V|} \quad \cos(\gamma) = \frac{V_Z}{|V|}$$

$$\vec{V} = |V|\cos(\alpha)\,\boldsymbol{i} + |V|\cos(\beta)\,\boldsymbol{j} + |V|\cos(\gamma)\,\boldsymbol{k}$$

$$cos^2(\alpha) + cos^2(\beta) + cos^2(\gamma) = 1$$

$$\hat{V} = U_V = \frac{V_X}{|V|}\boldsymbol{i} + \frac{V_Y}{|V|}\boldsymbol{j} + \frac{V_Z}{|V|}\boldsymbol{k}$$

$$\hat{V} = U_V = \cos(\alpha)\,\boldsymbol{i} + \cos(\beta)\,\boldsymbol{j} + \cos(\gamma)\,\boldsymbol{k}$$