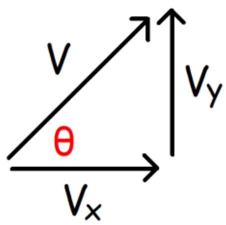
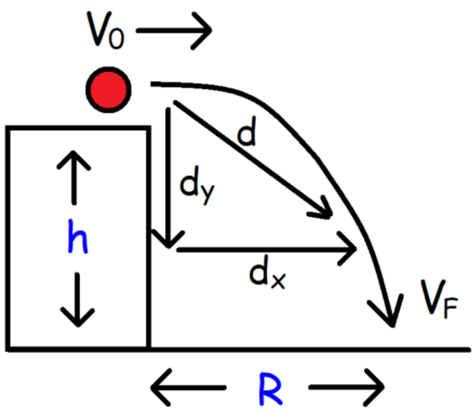
Projectile Motion Formula Sheet:



Vector Formulas:

$$\boldsymbol{v_x} = v\cos\theta \qquad \boldsymbol{v_y} = v\sin\theta$$

$$\boldsymbol{v} = \sqrt{v_x{}^2 + v_y{}^2} \qquad \boldsymbol{\theta} = tan^{-1}\left(\frac{v_y}{v_x}\right)$$



Facts to Know

1. $v_o = v_x = constant$
2. $v_{yo} = 0$
3. $a_y = -g = -9.8\ m/s^2$
4. $g = +9.8\ m/s^2$
5. $\theta = 0°\ and\ \cos 0° = 1$

Derived Equations:

Initial Velocity:

$$\boldsymbol{v_o} = R\sqrt{\frac{g}{2h}}$$

Range Vs Time:

$$\frac{R_1}{t_1} = \frac{R_2}{t_2}$$

Height of the Cliff/Building:

$$\boldsymbol{h} = \frac{1}{2}gt^2 \qquad h = \frac{R^2 g}{2v_o{}^2}$$

Range: $\cos 0° = 1\ and\ v_o = v_x$

$$\boldsymbol{R} = v_x t \qquad \boldsymbol{R} = v_o\cos\theta\, t \qquad \boldsymbol{R} = v_o\sqrt{\frac{2h}{g}}$$

Displacement:

$$\boldsymbol{d_x} = x_F - x_o \qquad \boldsymbol{d_y} = y_F - y_o$$

$$\boldsymbol{d_x} = v_o\cos\theta\, t \qquad \boldsymbol{d_y} = v_o\sin\theta\, t - \frac{1}{2}gt^2$$

$$\boldsymbol{d} = v_o t\sqrt{1 + \left(\frac{gt}{2v_o}\right)^2} \qquad \boldsymbol{d} = \sqrt{d_x{}^2 + d_y{}^2}$$

Final Vertical Velocity:

$$\boldsymbol{v_{yF}} = -gt$$

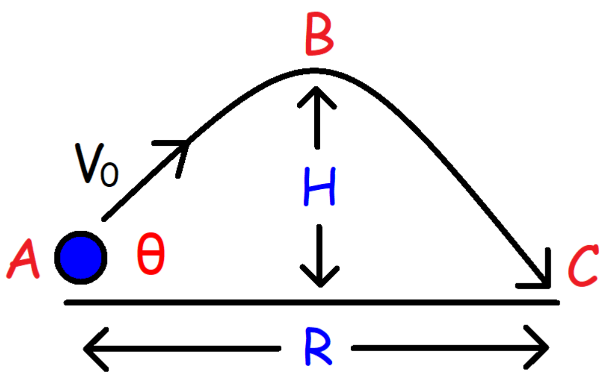
Final Velocity:

$$\boldsymbol{v_F} = \sqrt{v_o{}^2 + (gt)^2} \qquad \boldsymbol{v_F} = \sqrt{v_o{}^2 - 2gd_y} \qquad \boldsymbol{\theta_F} = cos^{-1}\left(\frac{v_o}{v_f}\right)$$

Note: *dy is negative when the projectile is falling. (g = +9.8)*

Time to Reach the Ground:

$$\boldsymbol{t} = \sqrt{\frac{2h}{g}} = \frac{R}{v_o} = \frac{v_{yF}}{-g} = \frac{\sqrt{v_F{}^2 - v_o{}^2}}{g}$$



Point A:

$$v_x = v_o \cos\theta \qquad v_{yo} = v_o \sin\theta$$

v_y *decreases by* 9.8 m/s *every second.*
v_x *is constant.*

Point B:

$$v_y = 0 \qquad v = v_x$$

All Points:

$$a_y = -g = -9.8\, m/s^2$$
$$g = +9.8\, m/s^2$$

Height:

$$H = \frac{{v_o}^2 \sin^2(\theta)}{2g} = \frac{R\tan\theta}{4} = \frac{{v_{yo}}^2}{2g}$$

$$H_{Max} = \frac{{v_o}^2}{2g} \quad when\ \theta = 90°$$

Range:

$$R = \frac{{v_o}^2 \sin(2\theta)}{g} = \frac{2v_x v_{yo}}{g} = \frac{4H}{\tan\theta}$$

$$R_{Max} = \frac{{v_o}^2}{g} \quad when\ \theta = 45°$$

Time of Flight:

$$t_{A\to B} = \frac{v_o \sin\theta}{g} \qquad t_{A\to C} = \frac{2v_o \sin\theta}{g}$$

Initial Angle:

$$\theta_o = tan^{-1}\left(\frac{4H}{R}\right) = sin^{-1}\left(\sqrt{\frac{2gH}{{v_o}^2}}\right) = \frac{1}{2} sin^{-1}\left(\frac{Rg}{{v_o}^2}\right)$$

Velocity Components:

$$v_x = v_o \cos\theta \qquad v_y = v_o \sin\theta - gt$$

Equation of Trajectory:

$$y = x\tan\theta - \frac{gx^2}{2{v_o}^2 cos^2(\theta)}$$

Note: $x = dx\ and\ y = dy\ if\ (x_o, y_o)\ is\ (0,0).$

Initial Velocity:

$$v_o = \sqrt{\frac{Rg}{\sin(2\theta)}} = \frac{\sqrt{2gH}}{\sin\theta} = \frac{g t_{A\to C}}{2\sin\theta}$$

Final Velocity:

$$v_F = \sqrt{{v_{xF}}^2 + {v_{yF}}^2}$$

$$v_F = \sqrt{{v_o}^2 - 2gtv_o \sin\theta + (gt)^2}$$

Position:

$$x_F = x_o + v_o \cos\theta\, t$$
$$y_F = y_0 + v_o \sin\theta\, t - \frac{1}{2} gt^2$$

Position Vector: $(x_o, y_o) \to (0,0)$

$$\vec{r} = [v_o \cos\theta\, t]\boldsymbol{i} + [v_o \sin\theta\, t - 1/2\, gt^2]\boldsymbol{j}$$

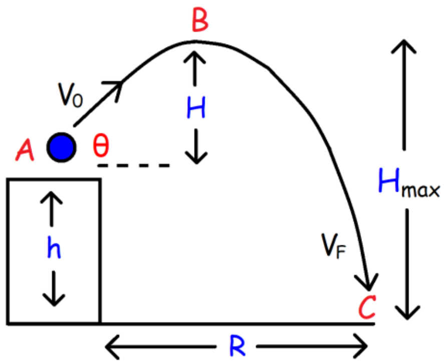
$$\vec{r} = r_x\boldsymbol{i} + r_y\boldsymbol{j} \qquad |\vec{r}| = \sqrt{{r_x}^2 + {r_y}^2}$$

Displacement:

$$d_x = v_o \cos\theta\, t \qquad d_y = v_o \sin\theta\, t - \frac{1}{2} gt^2$$

$$d = \sqrt{{d_x}^2 + {d_y}^2}$$

$$d = v_o t\sqrt{1 - \frac{gt\sin\theta}{v_o} + \left(\frac{gt}{2v_o}\right)^2} = |\vec{r}|$$



Point A:

$$\boldsymbol{v_x} = v_o \cos\theta \qquad \boldsymbol{v_{yo}} = v_o \sin\theta$$

v_y *decreases by* 9.8 m/s *every second.*

v_x *is constant.*

Point B:

$$v_y = 0 \qquad v = v_x$$

All Points:

$$a_y = -g = -9.8\, m/s^2$$

$$g = +9.8\, m/s^2 \text{ and } \boldsymbol{h = y_o}$$

Height:

$$H_{max} = h + H = y_o + \frac{v_o{}^2 sin^2(\theta)}{2g}$$

Range:

$$R = v_o \cos\theta\, t$$

Time of Flight:

$$t_{A\to B} = \frac{v_o \sin\theta}{g} \qquad t_{B\to C} = \sqrt{\frac{2H_{max}}{g}}$$

$$t_{A\to C} = \frac{v_o \sin\theta}{g} + \sqrt{\frac{2H_{max}}{g}}$$

$$t_{A\to C} = \frac{v_o \sin\theta + \sqrt{v_o{}^2 sin^2(\theta) + 2gy_o}}{g}$$

Displacement:

$$\boldsymbol{d_x} = v_o \cos\theta\, t \qquad \boldsymbol{d_y} = v_o \sin\theta\, t - \frac{1}{2}gt^2$$

$$\boldsymbol{d} = \sqrt{d_x{}^2 + d_y{}^2}$$

$$\boldsymbol{d} = v_o t\sqrt{1 - \frac{gt\sin\theta}{v_o} + \left(\frac{gt}{2v_o}\right)^2} = |\vec{r}|$$

$$\boldsymbol{d_x} = x_F - x_o \qquad \boldsymbol{d_y} = y_F - y_o$$

Equation of Trajectory:

$$\boldsymbol{y} = x\tan\theta - \frac{gx^2}{2v_o{}^2 cos^2(\theta)}$$

Note: $x = dx$ *and* $y = dy$ *if* (x_o, y_o) *is* $(0,0)$.

Position:

$$\boldsymbol{x_F} = x_o + v_o\cos\theta\, t$$

$$\boldsymbol{y_F} = y_0 + v_o \sin\theta\, t - \frac{1}{2}gt^2$$

Velocity Components:

$$\boldsymbol{v_x} = v_o\cos\theta \qquad \boldsymbol{v_y} = v_o \sin\theta - gt$$

Position Vector:

$$\vec{r} = [v_o\cos\theta\, t]\boldsymbol{i} + [v_o\sin\theta\, t - 1/2\, gt^2]\boldsymbol{j}$$

Initial Velocity:

$$v_o = \frac{R}{t\cos\theta} = \frac{\sqrt{2g(H_{max} - h)}}{\sin\theta} = \frac{1/2\, gt^2 - y_o}{t\sin\theta}$$

Initial Angle:

$$\boldsymbol{\theta_o} = cos^{-1}\left(\frac{R}{v_o t}\right) = sin^{-1}\left(\frac{\sqrt{2g(H_{max} - y_o)}}{v_o}\right)$$

$$\boldsymbol{\theta_o} = sin^{-1}\left(\frac{1/2\, gt^2 - y_o}{v_o t}\right) = tan^{-1}\left(\frac{1/2\, gt^2 - y_o}{R}\right)$$

Final Velocity:

$$\boldsymbol{v_F} = \sqrt{v_{xF}{}^2 + v_{yF}{}^2} \qquad \boldsymbol{\theta_F} = cos^{-1}\left(\frac{v_o\cos\theta_o}{v_f}\right)$$

$$\boldsymbol{v_F} = \sqrt{v_o{}^2 - 2gtv_o\sin\theta + (gt)^2} \qquad \boldsymbol{\theta_F} = tan^{-1}\left(\frac{v_y}{v_x}\right)$$

Height of the Building:

$$h = |d_y| = \left|v_o\sin\theta\, t - \frac{1}{2}gt^2\right|$$